Statistics With *Mathematica*'s

2.^1000

1.07151×10^{301}

st = {1., 2., 30.};

Mean[st]

11.

Variance[st]

271.

StandardDeviation[st]

16.4621

st = {1., 2., 30.};

Median[{{1}, {2}, {3}, {4}}]

$\left\{\frac{5}{2}\right\}$

Median[{{1}, {2}, {3}, {4}, {10}}]

{3}

To find 10 real Random numbers between 100 to 200, enter :

Table[Random[Real, {100, 200}], {10}]

{136.505, 173.074, 137.001, 120.377, 151.954, 194.979, 165.63, 159.011, 165.777, 128.202}

To get 20 random numbers between 0 and 1.0 enter :

Table[Random[Real, {0.0, 1.00}], {20}]

{0.812731, 0.267325, 0.67632, 0.0062329, 0.710442, 0.587537,
0.0509431, 0.500342, 0.429777, 0.0522408, 0.439064, 0.111137, 0.895136,
0.17832, 0.0740102, 0.380394, 0.525129, 0.974547, 0.554474, 0.430599}

Random[Complex, {-10 - 10 ⅈ, 10 + 10 ⅈ}]

-2.31272 + 0.45849 ⅈ

```
Table[Random[Complex, {-10 - 10 ⅈ, 10 + 10 ⅈ}], {10}]
```

```
{7.20598 - 4.29106 ⅈ, -5.01655 + 9.23261 ⅈ, 0.32222 + 7.91251 ⅈ,
 -1.44589 + 1.65457 ⅈ, -7.64674 - 7.60172 ⅈ, -8.14676 - 0.442551 ⅈ,
 -8.30044 - 1.9407 ⅈ, -4.90457 - 5.06524 ⅈ, -1.5128 + 4.59311 ⅈ, -7.22741 - 5.98895 ⅈ}
```

To obtain the correlation between two sets of variables “a”, and “b”, first activate the Stat. part.

```
<< Statistics`MultiDescriptiveStatistics`
```

```
a = {1232.`, 1115.`, 2205.`, 1897.`, 1932.`, 1612.`, 1598.`, 1804.`, 1752.`, 2067.`,
  2365.`, 1646.`, 1579.`, 1880.`, 1773.`, 1712.`, 1932.`, 1820.`, 1900.`, 2426.`,
  1558.`, 1470.`, 1858.`, 1587.`, 2208.`, 1487.`, 2206.`, 2332.`, 2540.`, 2322.`}
```

```
{1232., 1115., 2205., 1897., 1932., 1612., 1598., 1804., 1752., 2067.,
 2365., 1646., 1579., 1880., 1773., 1712., 1932., 1820., 1900., 2426.,
 1558., 1470., 1858., 1587., 2208., 1487., 2206., 2332., 2540., 2322.}
```

```
b = {4175.`, 6652.`, 7612.`, 10914.`, 10850.`, 7627.`, 6954.`, 8365.`, 9469.`, 6410.`,
  10327.`, 7320.`, 8196.`, 9709.`, 10370.`, 7749.`, 6818.`, 9307.`, 6457.`, 10102.`,
  7414.`, 7556.`, 7833.`, 8309.`, 9559.`, 6255.`, 10723.`, 5430.`, 12090.`, 10072.`}
```

```
{4175., 6652., 7612., 10914., 10850., 7627., 6954., 8365., 9469., 6410.,
 10327., 7320., 8196., 9709., 10370., 7749., 6818., 9307., 6457., 10102.,
 7414., 7556., 7833., 8309., 9559., 6255., 10723., 5430., 12090., 10072.}
```

```
Correlation[a, b]
```

```
0.549872
```

```
Correlation[a, b, ScaledMethod → MedianDeviation]
```

```
0.549872
```

********************Please note that the example below is from Freund stat. book page 311. He reports the answe as r=.92 ******************* *Mathematica* gives below: r=.924492

```
<< Statistics`MultiDescriptiveStatistics`
```

```
<< Statistics`MultiDescriptiveStatistics`
```

```
Clear[x, y];
```

```
x = {72., 70., 63., 74., 69., 72., 66., 68., 72., 70., 73., 71.}
```

```
{72., 70., 63., 74., 69., 72., 66., 68., 72., 70., 73., 71.}
```

```
y = {191., 172., 125., 210., 154., 186., 147., 159., 188., 164., 175., 163.}
```

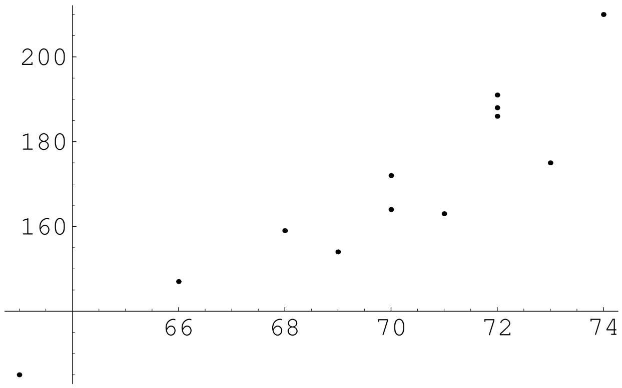
```
{191., 172., 125., 210., 154., 186., 147., 159., 188., 164., 175., 163.}
```

```
Correlation[x, y]
```

```
0.924492
```

```
dplot = {{72., 191.}, {70., 172.}, {63., 125.}, {74., 210.}, {69., 154.}, {72., 186.},
   {66., 147.}, {68., 159.}, { 72., 188.}, { 70., 164.}, {73., 175.}, {71. , 163.}};
```

```
plot1 = ListPlot[dplot]
```



- Graphics -

The above plot is also reported by Freund. ***************

```
Export["out1.dat", {6.7, 8.2, -5.3}, "List"]
```

out1.dat

```
Import["out1.dat"]
```

{{6.7}, {8.2}, {-5.3}}

```
data1 = Import["out1.dat"]
```

{{6.7}, {8.2}, {-5.3}}

```
Mean[data1]
```

{3.2}

```
data = {{0.055, 90}, {0.091, 97}, {0.138, 107}, {0.167, 124}, {0.182, 142},
   {0.211, 150}, {0.232, 172}, {0.248, 189}, {0.284, 209}, {0.351, 253}};
```

```
Mean[data]
```

$\left\{0.1959, \frac{1533}{10}\right\}$

```
Variance[data]
```

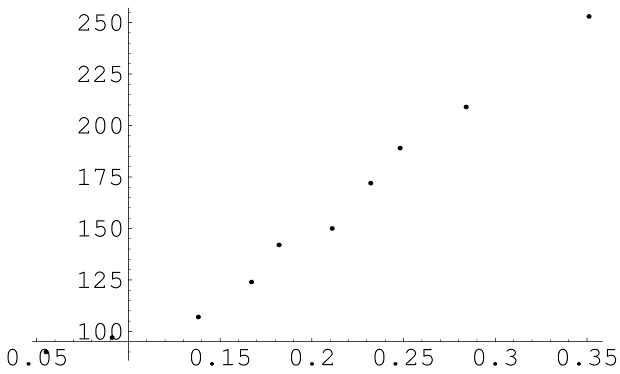
$\left\{0.00792232, \frac{249841}{90}\right\}$

```
<< Statistics`LinearRegression`
```

```
Clear[data]
```

```
data = {{0.055, 90}, {0.091, 97}, {0.138, 107}, {0.167, 124}, {0.182, 142},
   {0.211, 150}, {0.232, 172}, {0.248, 189}, {0.284, 209}, {0.351, 253}};
```

```
dplot = ListPlot[data]
```




```
- Graphics -
```

```
Datareg = {{12., 5.27}, {18., 5.68},
   {24., 6.25}, {30., 7.21}, {36., 8.02}, {42., 8.71}, {48., 8.42}};
```

```
Mean[Datareg]
```

```
{30., 7.08}
```

An Example of linear Regression from Freund Stat.book Pg.311.

```
<< Statistics`LinearRegression`
```

```
Clear[x, y]
```

```
x = {72, 70, 63, 74, 69, 72, 66, 68, 72, 70, 73, 71}
```

```
{72, 70, 63, 74, 69, 72, 66, 68, 72, 70, 73, 71}
```

```
y = {191, 172, 125, 210, 154, 186, 147, 159, 188, 164, 175, 163}
```

```
{191, 172, 125, 210, 154, 186, 147, 159, 188, 164, 175, 163}
```

```
Correlation[x, y]
```

```
Correlation[{72, 70, 63, 74, 69, 72, 66, 68, 72, 70, 73, 71},
 {191, 172, 125, 210, 154, 186, 147, 159, 188, 164, 175, 163}]
```

```
data = {{0.055, 90}, {0.091, 97}, {0.138, 107}, {0.167, 124}, {0.182, 142},
   {0.211, 150}, {0.232, 172}, {0.248, 189}, {0.284, 209}, {0.351, 253}};
```

```
Fit[data, {1, x}, x]
```

```
datareg0 = {{72, 191}, {70, 172}, {63, 125}, {74, 210}, {69, 154},
   {72, 186}, {66, 147}, {68, 159}, {72, 188}, {70, 164}, {73, 175}, {71, 163}};
```

```
<< Statistics`LinearRegression`
```

```
Fit[datareg0, {1, x}, x]
```

-299.111 + 6.69444 x

```
Datareg = {{12., 5.27}, {18., 5.68},
   {24., 6.25}, {30., 7.21}, {36., 8.02}, {42., 8.71}, {48., 8.42}};
```

```
Regress[Datareg, {1, x^2}, x]
```

{ParameterTable →

	Estimate	SE	TStat	PValue
1	5.37459	0.336652	15.9648	0.0000175548
x^2	0.00163353	0.000264182	6.18337	0.00161279

RSquared → 0.88435, AdjustedRSquared → 0.86122, EstimatedVariance → 0.260859,

ANOVATable →

	DF	SumOfSq	MeanSq	FRatio	PValue
Model	1	9.9737	9.9737	38.234	0.00161279
Error	5	1.3043	0.260859		
Total	6	11.278			

}

```
<< Statistics`ANOVA`
```

```
onewaydata = {{1, 7.0}, {1, 5.3}, {1, 5.9}, {1, 6.6}, {1, 4.9}, {2, 4.4},
   {2, 6.8}, {2, 7.7}, {2, 8.3}, {2, 6.6}, {3, 8.1}, {3, 10.4}, {3, 8.0},
   {3, 6.8}, {3, 9.2}, {4, 5.7}, {4, 3.9}, {4, 6.2}, {4, 5.5}, {4, 6.2}};
```

```
ANOVA[onewaydata]
```

{ANOVA →

	DF	SumOfSq	MeanSq	FRatio	PValue
Model	3	26.2935	8.7645	6.12795	0.0056161
Error	16	22.884	1.43025		
Total	19	49.1775			

,

CellMeans →

All	6.675
Model[1]	5.94
Model[2]	6.76
Model[3]	8.5
Model[4]	5.5

}

Below is an example AOV is from Page of 337 of Hole

```
onewaydata2 = {{1, 26.0}, {1, 28}, {1, 25}, {1, 23}, {1, 25}, {1, 27}, {1, 24},
   {1, 28}, {2, 26}, {2, 31}, {2, 27}, {2, 25}, {2, 28}, {2, 29}, {2, 26}, {2, 26},
   {3, 24}, {3, 27}, {3, 25}, {3, 26}, {3, 25}, {3, 23}, {3, 27}, {3, 25}};
```

```
ANOVA[onewaydata2]
```

{ANOVA →

	DF	SumOfSq	MeanSq	FRatio	PValue
Model	2	17.3333	8.66667	2.82171	0.0821504
Error	21	64.5	3.07143		
Total	23	81.8333			

,

CellMeans →

All	26.0833
Model[1]	25.75
Model[2]	27.25
Model[3]	25.25

}

This is an example of two way AOV,it comes from pag.345 of J.Freund.

```
twoway2 = {{1, 1, 58.2}, {1, 1, 52.6}, {1, 2, 56.2}, {1, 2, 41.2},
   {1, 3, 65.3} , {1, 3, 60.8}, {2, 1, 48.1}, {2, 1, 42.8}, {2, 2, 54.1},
   {2, 2, 50.5}, {2, 3, 51.6}, {2, 3, 48.4}, {3, 1, 60.1}, {3, 1, 58.3},
   {3, 2, 70.9}, {3, 2, 73.2}, {3, 3, 39.2}, {3, 3, 40.7}, {4, 1, 75.8},
   {4, 1, 71.5}, {4, 2, 58.2}, {4, 2, 51.0}, {4, 3, 48.7}, {4, 3, 41.4}};

ANOVA[twoway2, {factor1, factor2, All}, {factor1, factor2}]
```

{ANOVA →

	DF	SumOfSq	MeanSq	FRatio	PValue
factor1	3	272.95	90.9833	4.72334	0.0212153
factor2	2	363.948	181.974	9.44705	0.00343427
factor1 factor2	6	1782.99	297.165	15.4271	0.0000520171
Error	12	231.15	19.2625		
Total	23	2651.04			

,

CellMeans →

All	54.95
factor1[1]	55.7167
factor1[2]	49.25
factor1[3]	57.0667
factor1[4]	57.7667
factor2[1]	58.425
factor2[2]	56.9125
factor2[3]	49.5125
factor1[1] factor2[1]	55.4
factor1[1] factor2[2]	48.7
factor1[1] factor2[3]	63.05
factor1[2] factor2[1]	45.45
factor1[2] factor2[2]	52.3
factor1[2] factor2[3]	50.
factor1[3] factor2[1]	59.2
factor1[3] factor2[2]	72.05
factor1[3] factor2[3]	39.95
factor1[4] factor2[1]	73.65
factor1[4] factor2[2]	54.6
factor1[4] factor2[3]	45.05

}

```
onewaydatadun = {{1, 84.4}, {1, 116.0}, {1, 84.0}, {1, 68.6},
   {2, 64.4}, {2, 79.9}, {2, 88.0}, {2, 69.4}, {3, 75.2}, {3, 62.4}, {3, 62.4},
   {3, 73.8}, {4, 88.4}, {4, 90.2}, {4, 73.2}, {4, 87.8}, {5, 56.4}, {5, 83.2},
   {5, 90.4}, {5, 85.6}, {6, 65.6}, {6, 79.4}, {6, 65.6}, {6, 70.2}};
```

```
ANOVA[onewaydatadun, PostTests → Duncan]

                  DF     SumOfSq     MeanSq      FRatio      PValue
{ANOVA → Model    5      1246.28     249.256     1.66978     0.192859 ,
         Error    18     2686.95     149.275
         Total    23     3933.23

             All          77.6875
             Model[1]     88.25
             Model[2]     75.425
 CellMeans → Model[3]     68.45    , PostTests → {Model → Duncan  {}}}
             Model[4]     84.9
             Model[5]     78.9
             Model[6]     70.2
```

****************************** In the above NO sig. Diff is observed, and therefore the cells for Duncan test are empty. ********************

```
onewaydatadun2 = {{1, 19.4}, {1, 32.6}, {1, 27.0}, {1, 32.1},
   {1, 33.0}, {2, 17.7}, {2, 24.8}, {2, 27.9}, {2, 25.2}, {2, 24.3}, {3, 17.0},
   {3, 19.4}, {3, 9.1}, {3, 11.9}, {3, 15.8}, {4, 20.7}, {4, 21.0}, {4, 20.5},
   {4, 18.8}, {4, 18.6}, {5, 14.3}, {5, 14.4}, {5, 11.8}, {5, 11.6}, {5, 14.2}};

ANOVA[onewaydatadun2, PostTests → Duncan]

                  DF     SumOfSq     MeanSq      FRatio      PValue
{ANOVA → Model    4      838.598     209.649     15.3776     6.75994×10^-6 ,
         Error    20     272.668     13.6334
         Total    24     1111.27

             All          20.124
             Model[1]     28.82
             Model[2]     23.98
 CellMeans → Model[3]     14.64 ,
             Model[4]     19.92
             Model[5]     13.26

 PostTests → {Model → Duncan  {{1, 3}, {1, 4}, {1, 5}, {2, 3}, {2, 5}, {3, 4}, {4, 5}}}}
```

The above 2 examples of Duncan's test are from pages 105 and 101 of Steel and Torrie.

```
ANOVA[onewaydatadun2, PostTests → Tukey]
```

```
                  DF    SumOfSq   MeanSq    FRatio    PValue
{ANOVA →  Model   4     838.598   209.649   15.3776   6.75994×10^-6 ,
          Error   20    272.668   13.6334
          Total   24    1111.27

             All         20.124
             Model[1]    28.82
             Model[2]    23.98
 CellMeans → Model[3]    14.64 ,
             Model[4]    19.92
             Model[5]    13.26

 PostTests → {Model → Tukey  {{1, 3}, {2, 3}, {1, 4}, {1, 5}, {2, 5}}}}
```

****************Note that the Tukey's test and Duncan's test yield different answers. Care should be taken as some tests are more conservative that others. **********************

**************************Below is an example of non−linear fit ******************

```
datanon = {{1., 2.7}, {2, 7.3}, {2.5, 12.0}};

<< Statistics`NonlinearFit`

NonlinearFit[datanon, Exp[x], {x}, RegressionReport → BestFitParameters]

Clear[x, a]

datanon = {{1., 2.7}, {2, 7.3}, {2.5, 12.0}};

NonlinearFit[datanon, Exp[a x], {x}, {a}, RegressionReport → BestFitParameters]
```

$e^{0.993953\,x}$

```
Clear[a, x]

datanon = {{1., 2.9}, {2, 7.35}, {2.5, 12.1}};

datanon = {{1., 2.9}, {2, 7.35}, {2.5, 12.1}};
```

$e^{0.997745\,x}$

```
datanon1 = {{1., 1.}, {2, 3.90}, {2.5, 6.09}};

NonlinearFit[datanon, x^3 a, {x}, {a}, RegressionReport → BestFitParameters]
```

$0.81116\,x^3$

NonlinearFit[datanon, x^{2a}, {x}, {a}, RegressionReport → BestFitParameters]

$x^{2.74467}$

```
NonlinearRegress[datanon, x^(2 a), {x}, {a}]

{BestFitParameters → {a → 1.37233},
 ParameterCITable →    Estimate   Asymptotic SE   CI
                     a 1.37233    0.0584595       {1.1208, 1.62386}',
 EstimatedVariance → 2.04998,
                                         DF   SumOfSq   MeanSq
               Model                     1    204.743   204.743
 ANOVATable →  Error                     2    4.09996   2.04998,
               Uncorrected Total         3    208.842
               Corrected Total           2    42.335
 AsymptoticCorrelationMatrix → ( 1. ),
                                                   Curvature
                      Max Intrinsic                0.00915775
 FitCurvatureTable →  Max Parameter-Effects        1.02907 × 10^-17 }
                      95. % Confidence Region      0.232415
```

********************Data below comes from page 507 and 563 of Stat book by Alan H. Kvanli. THis data is chosen to show how to pick various columns of the data to run different statistics on the, E.g. choosing variable 4 and variable 1 in the bracket to perform linear regression. ****************

```
datalinreg = {{22., 2., 4., 16.}, {26., 2. , 8., 17.},
   {45., 3., 7., 26.}, {37., 4., 0.0, 24.}, {28., 4., 2., 22}, {50., 3., 10, 21.},
   {56., 6., 8., 32.}, {34., 3., 8., 18.}, {60., 5., 2., 30.}, {40., 3., 6., 20}};

Reverse /@ datalinreg[[All, {1, 4}]]

{{16., 22.}, {17., 26.}, {26., 45.}, {24., 37.},
 {22, 28.}, {21., 50.}, {32., 56.}, {18., 34.}, {30., 60.}, {20, 40.}}

Fit[Reverse /@ datalinreg[[All, {1, 4}]], x, x]

1.77318 x

<< Statistics`MultiDescriptiveStatistics`

Correlation[Flatten[datalinreg[[All, {1}]]], Flatten[datalinreg[[All, {2}]]]]

0.721252

Correlation[Flatten[datalinreg[[All, {1}]]], Flatten[datalinreg[[All, {4}]]]]

0.843254
```

You'll find the above result ".8432.." in the middle of page 507 of Kvanli which is the sample corr. coefficient between the first and the last variable in the bracket, e.g. {22., ..., 16}.

```
Regress[Flatten[datalinreg[[All, {1}]]], Flatten[datalinreg[[All, {4}]]], {1, x}, x]

Regress[{22., 26., 45., 37., 28., 50., 56., 34., 60., 40.},
 {16., 17., 26., 24., 22, 21., 32., 18., 30., 20}, {1, x}, x]

<< Statistics`LinearRegression`
```

```
Regress[Flatten[datalinreg[[All, {1}]]], Flatten[datalinreg[[All, {4}]]], {1, x}, x]
```

```
Regress[{22., 26., 45., 37., 28., 50., 56., 34., 60., 40.},
 {16., 17., 26., 24., 22, 21., 32., 18., 30., 20}, {1, x}, x]
```

```
Correlation[Flatten[datalinreg[[All, {1}]]], Flatten[datalinreg[[All, {4}]]]]
```

```
0.843254
```

```
Regress[Flatten[datalinreg[[All, {1}]]], Flatten[datalinreg[[All, {4}]]], {1, x}, x]
```

```
Regress[{22., 26., 45., 37., 28., 50., 56., 34., 60., 40.},
 {16., 17., 26., 24., 22, 21., 32., 18., 30., 20}, {1, x}, x]
```

```
<< Statistics`LinearRegression`
```

```
Regress[Flatten[datalinreg[[All, {1}]]], Flatten[datalinreg[[All, {4}]]], {1, x}, x]
```

```
Regress[{22., 26., 45., 37., 28., 50., 56., 34., 60., 40.},
 {16., 17., 26., 24., 22, 21., 32., 18., 30., 20}, {1, x}, x]
```

```
Fit[Reverse /@ datalinreg[[All, {1, 2}]], x, x]
```

```
10.9416 x
```

```
10.941605839416058` x
```

```
Fit[Reverse /@ datalinreg[[All, {4, 1}]], {1, x}, x]
```

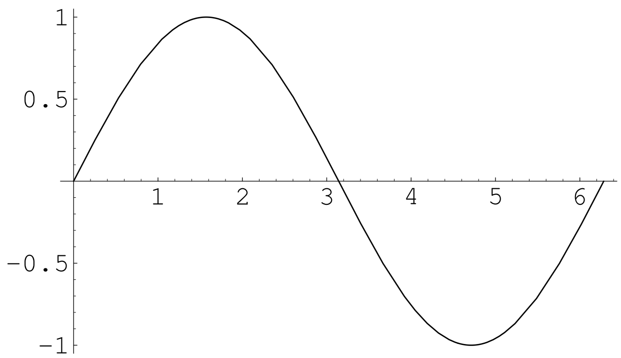
```
8.51396 + 0.353921 x
```

***********Note the above Regression equation is the same the regression equation obtained on page of 511 of Kvanli book*****

```
Print["f[x]=Sin[x]"]
```

```
f[x]=Sin[x]
```

```
Plot[f[x], {x, 0, 2 π}]
```



- Graphics -

```
data = {{0.055, 90}, {0.091, 97}, {0.138, 107}, {0.167, 124}, {0.182, 142},
   {0.211, 150}, {0.232, 172}, {0.248, 189}, {0.284, 209}, {0.351, 253}};
```

```
<< Statistics`LinearRegression`
```

```
Fit[data, {1, x}, x]
```

39.6484 + 580.151 x

```
Datareg = {{12., 5.27}, {18., 5.68},
   {24., 6.25}, {30., 7.21}, {36., 8.02}, {42., 8.71}, {48., 8.42}};
```

```
Regress[Datareg, {1, x^2}, x]
```

{ParameterTable →

	Estimate	SE	TStat	PValue
1	5.37459	0.336652	15.9648	0.0000175548
x^2	0.00163353	0.000264182	6.18337	0.00161279

, RSquared → 0.88435, AdjustedRSquared → 0.86122, EstimatedVariance → 0.260859,

ANOVATable →

	DF	SumOfSq	MeanSq	FRatio	PValue
Model	1	9.9737	9.9737	38.234	0.00161279
Error	5	1.3043	0.260859		
Total	6	11.278			

}

```
<< Statistics`MultiDescriptiveStatistics`
```

```
a1 = {1, 2, 3}
```

{1, 2, 3}

```
b1 = {1.1, 2.0, 3.01}
```

{1.1, 2., 3.01}

```
Correlation[a1, b1]
```

Correlation[{1, 2, 3}, {1.1, 2., 3.01}]

An Example of linear correlation from Stat.bookby Freund, Pg .311.

```
Clear[x, y]
```

```
x = {72, 70, 63, 74, 69, 72, 66, 68, 72, 70, 73, 71}

{72, 70, 63, 74, 69, 72, 66, 68, 72, 70, 73, 71}

y = {191, 172, 125, 210, 154, 186, 147, 159, 188, 164, 175, 163}

{191, 172, 125, 210, 154, 186, 147, 159, 188, 164, 175, 163}

Correlation[x, y]

Correlation[{72, 70, 63, 74, 69, 72, 66, 68, 72, 70, 73, 71},
 {191, 172, 125, 210, 154, 186, 147, 159, 188, 164, 175, 163}]
```

$$\frac{241}{2\sqrt{16989}}$$

$$\frac{241}{2\sqrt{16989}}$$

$$N\left[\frac{241}{2\sqrt{16989}}\right]$$

```
0.924492
```

Example of One Way AOV from Pg.331 of Stat Book Freund

```
<< Statistics`ANOVA`

onewaydata = {{1, 73.0}, {1, 57.}, {1, 95.}, {1, 78.}, {1, 86.}, {1, 61.}, {1, 80.},
   {1, 98.}, {1, 64.}, {1, 78.}, {2, 84.}, {2, 95.}, {2, 96.}, {2, 62.}, {2, 80.},
   {2, 87.}, {2, 100.}, {2, 74.}, {2, 85.}, {2, 77}, {3, 69.}, {3, 80}, {3, 73.},
   {3, 62.}, {3, 50}, {3, 71}, {3, 84}, {3, 66}, {3, 52}, {3, 73}, {4, 65.}, {4, 58.},
   {4, 82.}, {4, 86.}, {4, 35.}, {4, 52.}, {4, 70.}, {4, 79.}, {4, 43.}, {4, 60}};

ANOVA[onewaydata]
```

{ANOVA →

	DF	SumOfSq	MeanSq	FRatio	PValue
Model	3	2620.	873.333	4.8399	0.00624316
Error	36	6496.	180.444		
Total	39	9116.			

,

CellMeans →

All	73.
Model[1]	77.
Model[2]	84.
Model[3]	68.
Model[4]	63.

}

```
Mean[onewaydata]
```

$\left\{\frac{5}{2}, 73.\right\}$

To obtain Mode the "DescriptiveStatistics`" needs to be activated. Under Mathematica's kernel Mode is defined a bit different.

```
<< Statistics`DescriptiveStatistics`
```

```
mean = {1., 2., 2., 2., 4., 4., 4., 5., 8.}
```

```
{1., 2., 2., 2., 4., 4., 4., 5., 8.}
```

```
Mode[mean]
```

```
{2., 4.}
```